

GeoGebra Basics

Handbook 9

A Guide for Learning GeoGebra

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Dilation using GeoGebra

A dilation is determined by a center and a number. For example, if the center is the origin and the number is $1/2$, then each point (x, y) gets transformed to the point $(x/2, y/2)$. The number can be a typed-in scale factor or a Marked Ratio determined by selecting two segments. Letting n denote the length of the first segment and d the length of the second, the Marked Ratio is n/d .

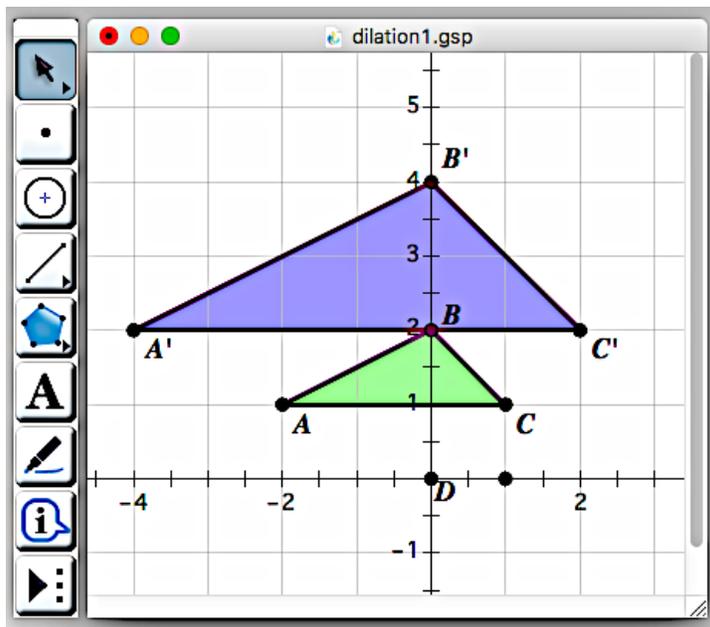


Deepening your Understanding of Dilations

A dilation is another type of transformation. A dilation can be either a reduction or an enlargement. In a dilation, each point of a geometric figure moves so that its distance from the center of dilation is multiplied by a scale factor (k).

$\triangle A'B'C'$ is the image of its' preimage $\triangle ABC$ under a dilation with center $(0,0)$ and scale factor 2. When the center of dilation is the origin, the coordinates of each point of the original figure are multiplied by the scale factor. For example, $A = (-2,1)$, so $A' = (2 \cdot -2, 2 \cdot 1) = (-4, 2)$. Refer to figure 117 below.

Figure 117 – dilation 2



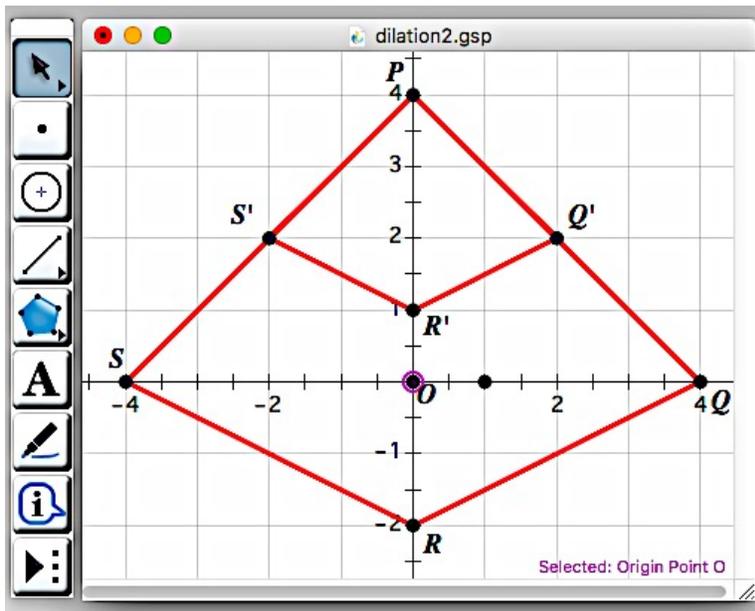
When the scale factor is greater than 1, the dilation is an enlargement. When the scale factor is between 0 and 1, the dilation is a reduction.

$P'Q'R'S'$ is the image of preimage PQRS under a dilation with center P and scale factor $\frac{1}{2}$. When the center of dilation is not the origin, the distance from P to each point of the figure is multiplied by $\frac{1}{2}$. Point P does not move. Refer to figure 118 below.

Figure 118 – dilation 3

For example, the distance from P to S is 4 units to the left and 4 units down. So, the distance from P to S is $\frac{1}{2} \cdot 4$ left and $\frac{1}{2} \cdot 4$ down, or 2 left and 2 down, and S is located at (-2,2).

A dilation produces a figure that is similar but not congruent to the original figure. Angle measures do not change, but side lengths are multiplied by the scale factor.



Things to Remember about Dilations

A dilation is sometimes called a similarity transformation.

Dilation notation:

$$D_{X,k}(F) = F'$$

F' is the image of Figure F under a dilation with center X and scale factor k . Many dilations are centered at the origin, and can be written as follows:

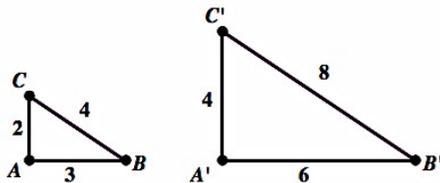
$$D_k(F) = F', \text{ here } k \text{ is scale factor.}$$

A dilation centered at the origin changes coordinates of point as follows:

$$D_k(x,y) = F(kx,ky)$$

Similar figures have the same shape, but not the same size. The angles are congruent, and the side lengths are proportional. Refer to figure 119 below.

Figure 119 – dilation 4



$$\angle A \cong \angle A', \angle B \cong \angle B', \\ \text{and } \angle C \cong \angle C'.$$

$$\frac{AB}{A'B'} = \frac{AC}{A'C'} = \frac{BC}{B'C'}$$

Cross multiply to solve proportions. Refer to figure 120 below.

$$\frac{x}{8} = \frac{2}{4} \quad 4x = 16 \quad x = 4$$

Figure 120 – dilation 5

Test Your Understanding of Dilation

- The diagram shows the result of a dilation of $\triangle PQR$ centered at the origin. What is the scale factor of this dilation? Refer to figure 121 below.

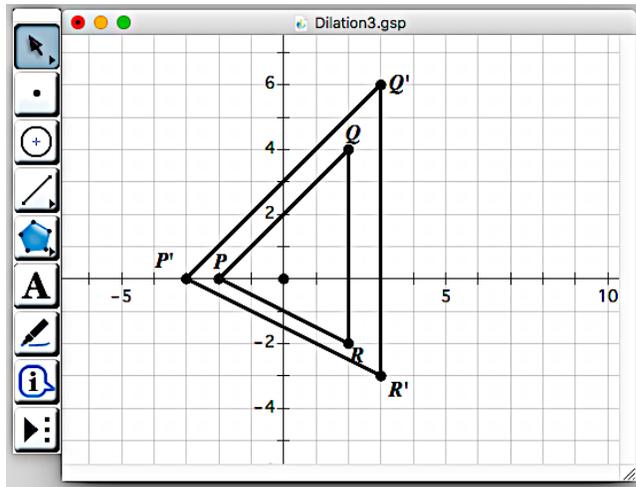


Figure 121 – dilation 6

- Use the diagram below to answer the next two questions. In the diagram, $\triangle R'S'T'$ will be the image of its' preimage $\triangle RST$ under a dilation with scale factor of $1/3$, centered at point R. Refer to figure 122 below.

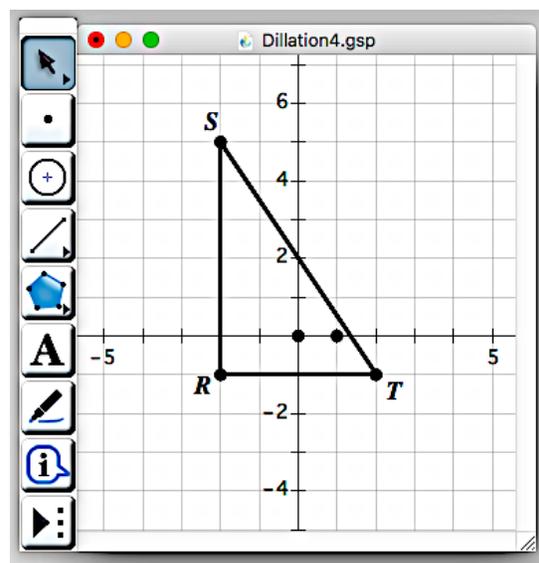


Figure 122 – dilation 7

3. Which best describes this dilation?

- a. $D_{1/3}(\triangle RST) = (\triangle R'S'T')$
- b. $D_{1/3}(\triangle R'S'T') = (\triangle RST)$
- c. $D_{r,1/3}(\triangle RST) = (\triangle R'S'T')$
- d. $D_{r,1/3}(\triangle R'S'T') = (\triangle RST)$

- 1. What will be the coordinates of points R'S'T' ?
- 2. After the dilation, which statement will be true?

a. $m\angle S = m\angle T$

b. $m\angle S' = \frac{1}{3}m\angle S$

c. $\overline{RS} = \overline{S'T}$

d. $R'S' = \frac{1}{3}RS$

Composites

You have now experimented with each of the four elementary transformations—translation, rotation, dilation, and reflection—we are in a position to view these as “elements” with which to compose “compounds”. For example, a compound transformation called glide reflection occurs when a reflection is followed by a translation in a direction parallel to the mirror.

Three of the basic transformations carry objects onto images that are congruent to the originals: in other words, these transformations preserve distance. The only basic transformation that does not preserve distance (unless the ratio used is ± 1) is dilation.

The Latin roots for “same distance” being essentially “iso” and “metric”, it is natural to describe as isometric these three transformations: translation, rotation, and reflection. An important theorem states that composites of these are also isometric. That is, if you use any number of these three elementary transformations, in any order, the resulting composite transformation is an isometry. As a converse of the theorem, every isometry is a composite of at most three reflections. Sketch the triangle having vertices $A = (0,0)$ $B = (4,0)$ $C = (0,3)$

1. Figure out a composite of elementary transformations that carries triangle ABC onto the triangle T that has vertices $(5, 3)$, $(5, 6)$, $(9, 6)$. Refer to figure 123 below.

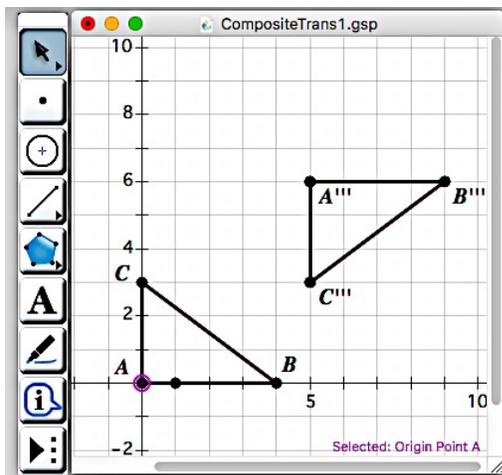


Figure 123 – Composite 1

Hint: Sketch the image, then translate, then reflect.

2. Create a sketch that confirms the following proposition: the composite of three reflections in three concurrent lines is a reflection in a line that passes

through the point of concurrence. (Note: a set of lines or curves are said to be concurrent if they all intersect at the same point). Refer to figure 124 below.

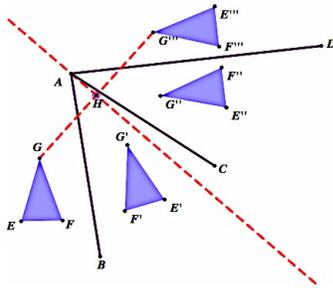


Figure 124 – Composite 2

3. Create a sketch that confirms the following proposition: if two objects are congruent, then there exists a composite of three elementary isometries that transforms one object into the other.
4. Try this: Create object $A'B'C'D'$ congruent to object $ABCD$. Then, (1) translate $ABCD$ by vector AA' ; (2) rotate AB' so that vector $A'B' =$ vector AB ; (3) if needed, reflect the image-object in line $A'B'$.

Similarity

Two geometric figures are congruent if each can be obtained from the other by an isometry. But, as you know, sometimes figures have identical shapes without having identical sizes—then they are similar rather than congruent. A major theorem states that two figures are similar if one can be obtained from the other by an isometry followed by a dilation.

Working with Matrices

An arbitrary fully determined system of independent linear equations, for example

$$2x + 3y + 2z = 3$$

$$x + y + z = 2$$

$$-y + 3z = 7$$

can be written as a matrix multiplication, namely

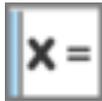
$$\underbrace{\begin{pmatrix} 2 & 3 & 2 \\ 1 & 1 & 1 \\ 0 & -1 & 3 \end{pmatrix}}_A \cdot \underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_X = \underbrace{\begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix}}_B.$$

In general, this permits us to consider the simple problem of solving the single matrix equation $A \cdot X = B$ instead. This in turn can be achieved by multiplying with the inverse matrix of A from the left.

Preparations

- Open a new GeoGebra window.

- Switch to Perspectives –



Construction Steps

Finishing your construction, the CAS View will look like this:

T 0 0 7 0 0 4 0 2 1	
1 <input type="radio"/>	A := {{2, 3, 2}, {1, 1, 1}, {0, -1, 3}} → $\mathbf{A} := \begin{pmatrix} 2 & 3 & 2 \\ 1 & 1 & 1 \\ 0 & -1 & 3 \end{pmatrix}$
2 <input type="radio"/>	B := {{3}, {2}, {7}} → $\mathbf{B} := \begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix}$
3 <input type="radio"/>	Invert[A] * B → $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$

1 Enter the coefficient matrix A as $A := \{\{2, 3, 2\}, \{1, 1, 1\}, \{0, -1, 3\}\}$

2 Define the column vector B as $B := \{\{3\}, \{2\}, \{7\}\}$

3 Calculate the result via $\text{Invert}[A] * B$

Hint: Matrices are entered as a list of their rows from top to bottom. The rows themselves are provided as lists containing the actual values from left to right.

Therefore, the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is given as $\{\{a, b\}, \{c, d\}\}$.

Using GeoGebra's Computer Algebra System (CAS) to Manipulate Equations

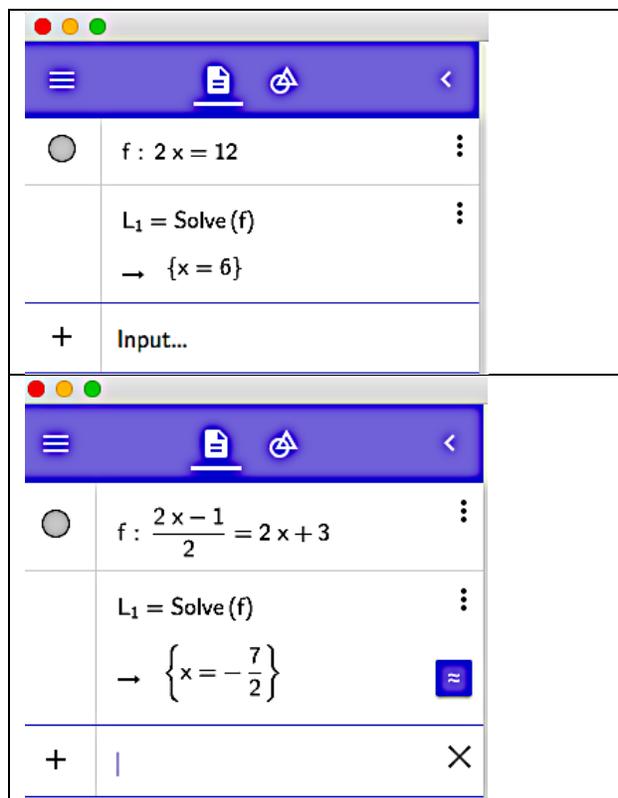
Manipulating terms and equations are important topics in middle school mathematics classes. Below is an example of how CAS can help solve algebra problems.

Open a new GeoGebra window.

Switch to the input window - 

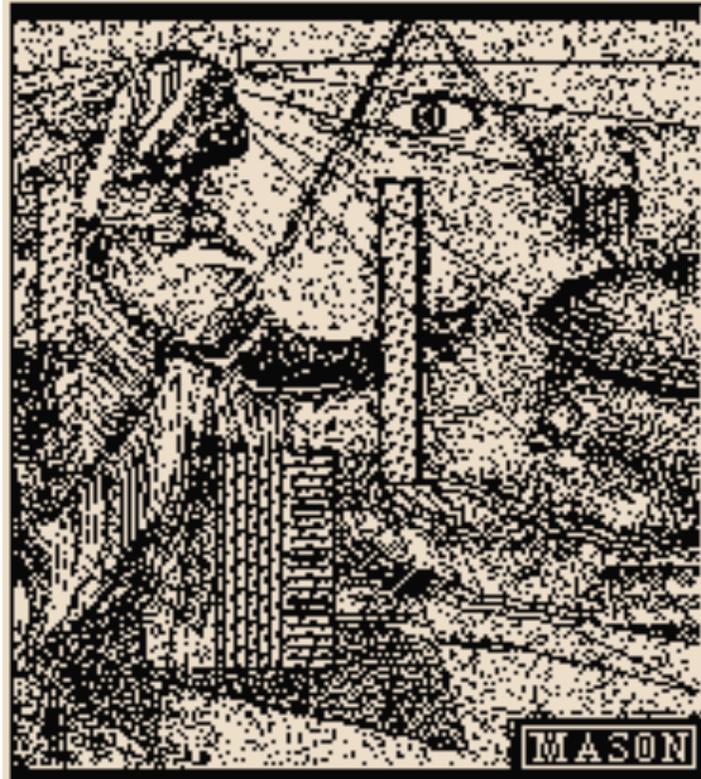
Introduction of new Tools

	Evaluate
	Numeric



The first screenshot shows the input window with the equation $f : 2x = 12$. Below it, the command $L_1 = \text{Solve}(f)$ is entered, resulting in the solution $\rightarrow \{x = 6\}$. The second screenshot shows the equation $f : \frac{2x-1}{2} = 2x+3$. The command $L_1 = \text{Solve}(f)$ is entered, resulting in the solution $\rightarrow \left\{x = -\frac{7}{2}\right\}$. A numeric button is visible next to the solution in the second screenshot.

<p> $a = \text{GCD}(240, 160)$ $\rightarrow 80$ </p> <p>Input...</p>	<p> $\text{GCD}(240, 160)$ $\rightarrow 80$ </p> <p> $\text{GCD}(\langle \text{List of Numbers} \rangle)$ $\text{GCD}(\langle \text{Number} \rangle, \langle \text{Number} \rangle)$ </p>
<p> $\text{LCM}(240, 160)$ $\rightarrow 480$ </p>	<p> LCM </p> <p> $\text{LCM}(\langle \text{List of Numbers} \rangle)$ CREATE SLIDER $\text{LCM}(\langle \text{Number} \rangle, \langle \text{Number} \rangle)$ </p>
<p> $\text{Factors}(x^2 - 5x + 6)$ $\rightarrow \{\{x - 3, 1\}, \{x - 2, 1\}\}$ </p>	<p> Factor </p> <p> $\text{Factor}(\langle \text{Polynomial} \rangle)$ CREATE SLIDER $\text{Factors}(\langle \text{Polynomial} \rangle)$ $\text{Factors}(\langle \text{Number} \rangle)$ </p>
<p> $\text{Factors}(240)$ $\rightarrow \{\{2, 4\}, \{3, 1\}, \{5, 1\}\}$ </p>	



I hope that you have learned many things about yourself as a learner, Geometry, and GeoGebra while on this mathematical journey. We covered only the tip of the iceberg in terms of what is possible with GeoGebra and geometry. Hopefully your experiences have sparked a new curiosity about mathematics within you, and you will study advanced topics in mathematics in the future.