City Smart Scholars
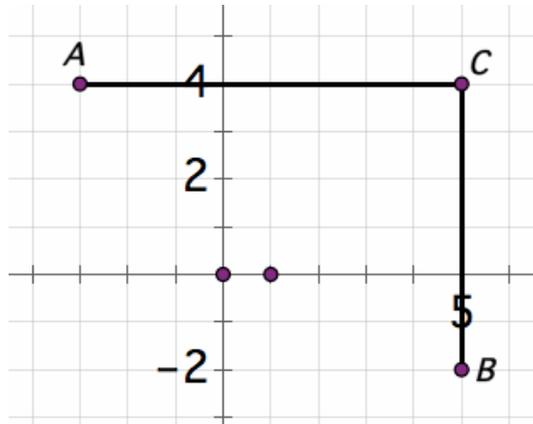
Activity Sheet 1: Deriving the Distance Formula

1. Use the diagram below to answer the following questions. Record your work in your toolbox-book.

a. What is AC?

b. What are the coordinates of A and C?

c. Use the coordinates of A and C to compute AC. Show your work.

d. What are the coordinates of C and B?

e. Use the coordinates of C and B to compute CB. Show your work.

f. Draw the AB. What kind of triangle is ABC? For that kind of triangle, what are AC and CB Called? What is AB called? Remember the distinction between declarative knowledge, procedural knowledge, and conceptual knowledge.

g. What theorem can you use to find the length of the hypothenuse (new vocabulary) of a right triangle if you know the lengths of the two legs?

h. Use your answer to g (above) to find AB.

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2. Use the diagram below to answer the following questions.
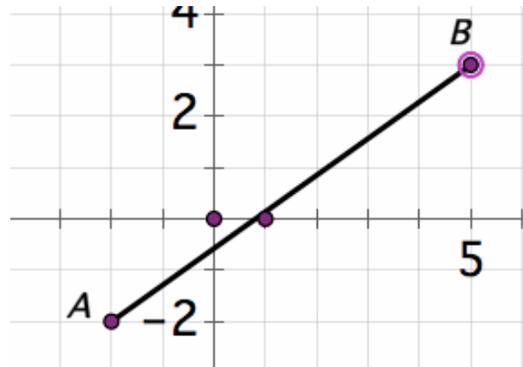
a. On the diagram above, create a right triangle with a horizontal leg, vertical leg, and hypothenuse AB. Label the vertex (vocabulary) of the right triangle C. Is this the only right triangle you could have drawn?
b. Find AC and CB.
c. Use Ac and CB to find AB.

3. The endpoints of a vertical segment AB and $A(x_1, y_1)$ and $B(x_2, y_2)$. Use the diagram for the following questions. (Do not count. Graph is not to scale.)
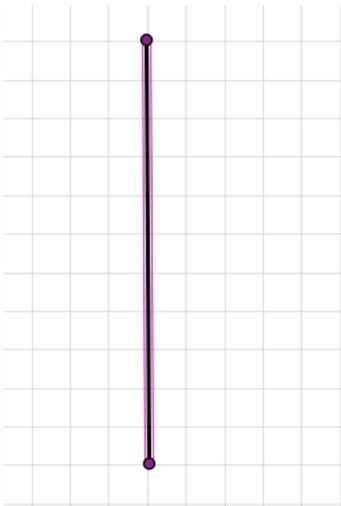
a. Label the lower point $A(x_1, y_1)$ and the upper point $B(x_2, y_2)$. Since AB is a vertical segment, what can you say about x1 and x2?
b. Express AB in terms of y_1 and y_2.
c. Why is it necessary to use absolute value (declarative knowledge) for the formulas above?
d. Does it matter which of the two formulas above you use?
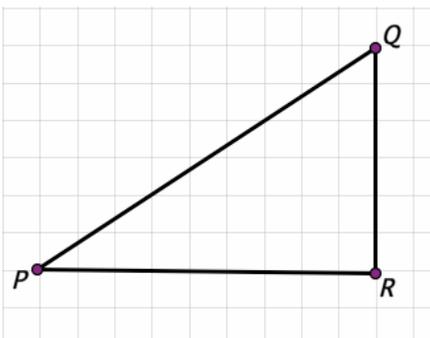e. Write a formula for AB using either formula above.

f. AB =

g. The endpoints of a vertical segment are G(–10, 12) and H(–10, –22). Use one of your formulas to compute GH.

4. The endpoints of a horizontal segment are C(x1, y2) and D(x2, y2). Use the diagram below for the following questions. (Do not count. Graph is not to scale.)

a. Label the point on the left C(x1, y1) and the point on the right D(x2, y2). Since CD is a horizontal segment, what can you say about y1 and y2?

b. Express CD in two different ways in terms of X1 and x2.

c. Write a formula for CD using either formula above.

CD=

d. The endpoints of a horizontal segment are E(–10, 12) and F(24, 12). Use your formula to compute EF.

5. The endpoints of a segment PQ are $P(x_1, y_1)$ and $Q(x_2, y2)$. Use the diagram below for the following questions. (Do Not count. Graph is not to scale.)

a. Use the Pythagorean Theorem on the right triangle, using PQ, PR, and QR. Complete the equation below.

$(PQ)^2 =$

b. Label the point P as (x_1, y_1) and point Q as (x_2, y_2). Find the coordinates of the point R.

c. Write formulas for PR and QR using either formula above.

PR =

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QR =

d. Determine whether this equation is true. If it is true, explain why. If it is false, give a counterexample. (Hint: when you square a real number (look this concept up), is it ever negative?)

$|a|^2 = a^2$

e. Use the last three problems to get a formula for $(PQ)^2$ in terms of x_1, x_2, y_1, and y_2.

$(PQ)^2$

f. Take the square root of both sides of your last formula to write PQ in terms of x_1, x_2, y_1, and y_2.

PQ =

g. The endpoints of a segment are i(8, 12) and J(2, 4). Use the formula you found to compute the distance between I and J.

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# Activity Sheet 2: Deriving the Midpoint Formula

6. The midpoint of a segment is the point on the segment that is the same distance from both endpoints. Use the graph below to answer the following question. Write you answers in your toolbox.
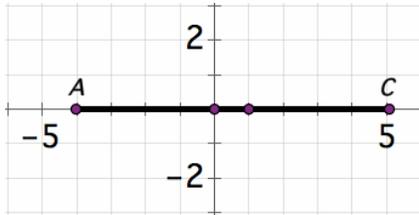
i. What are the x-coordinates of A and C?
j. What is the average of the x-coordinates of A and C?
k. What number is halfway between the x-coordinates of A and C?
l. What are the coordinates of the midpoint of AC? Graph and label the midpoint.
m. Explain the relationship among the answers to questions b, c, and d.

7. Use the graph below to answer the following questions.
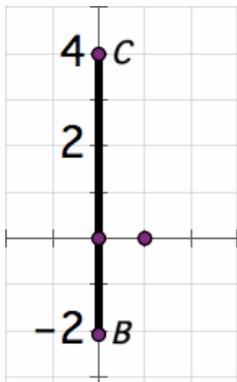
d. What are the y-coordinates of B and C?
e. What is the average of the y-coordinates of B and C?
f. What number is halfway between the y-coordinates of B and C?
g. What are the coordinates of the midpoint of BC? Graph and label the midpoint.
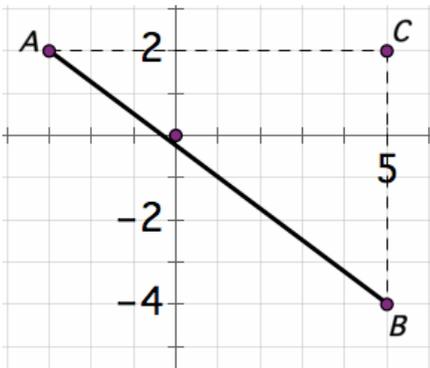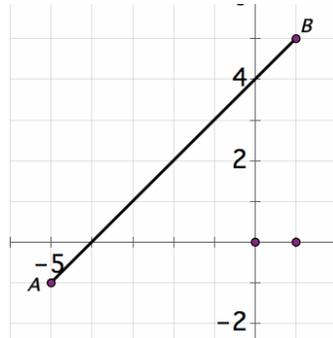h. Explain the relationship among the answers to questions b, c, and d.

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8. Use the diagram below to answer the following questions


h. Graph the midpoint AC. Label it *P*. What is the x-coordinate of this point?
i. What is the average of the x-coordinates of A and C? How is this related to your answer to a?
j. Graph the midpoint of BC. Label it *Q*. What is the y-coordinate of this point?
k. What is the average of the y-coordinates of B and C? How is this related to your answer to c?

l. What is the average of the x-coordinates of A and B?
m. What is the average of the y-coordinates of A and B?
n. What is the midpoint of BC?
o. How is the midpoint of AB related to the answers to 3e and 3f?

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9. Use the diagram below to answer the following questions.

e. What is the average of the x-coordinates of A and B?
f. What is the average of the y-coordinates of A and B?
g. What the coordinates of the midpoint of AB?
h. Explain the relationships among the answers to questions b, c, and d.
10. The endpoints of a segment PQ are $P(x_1, y_1)$ and $Q(x_2, y2)$. Use the diagram below for the following questions. (Do Not count. Graph is not to scale.)
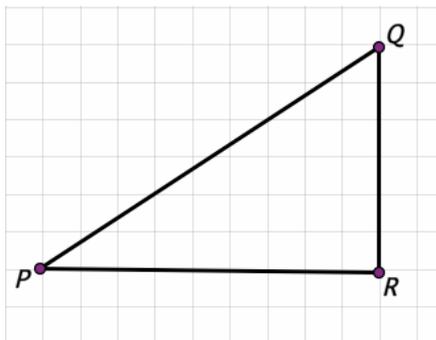
h. Label the point P as (x_1, y_1) and point Q as (x_2, y_2).
i. Write a formula for the average of the x-coordinates A and B.
j. Write a formula for the average of the y-coordinates A and B.
k. One way to think of the midpoint of PQ is as follows: average of the x-coordinates, average of the y-coordinates. Use this to derive a formula for the midpoint of PQ
l. The endpoints of a segment are E(8,12) and F(2,4). Use your formula to compute the midpoint of EF.

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Classwork/Homework

1. Find the length of the segment AB given the following:

(a) A(3,4), B(0,0) (b) A(5,4), B(7,9) (c) A(5,4), B(5,5)
(d) A(-9,-1), B(-7,1)

2. Find the coordinates of the midpoint of each of the segments given in Problem 1.

3. Find the lengths of the sides of the triangle whose vertices are given and construct the triangle of Geometer's Sketchpad (GSP).
    (a) A(2,3), B(4,-1), C(7,2)

4. Use the converse of the Pythagorean theorem to determine whether triangle ABC is a right triangle.
    (a) A(3,5), B(6,4), C(0, -4)
    (b) A(2,2), B(6,4), C(5,6)

5. Suppose we are given a quadrilateral ABCD with coordinates A(0,0), B(4,1), C(7,4), and D(3,3). Show that m(AB) = m(DC) and m(AD = m(BC).

6. Given A(0,7), B(-2,-1), C(2,-2), and D(4,6). Show that quadrilateral ABCD is a parallelogram.